Numerical Simulation of Fluid Flow and Geomechanics

Liuqi Wang[1], Humid Roshan[2] & Rick Causebrook[1],

[1]Geoscience Australia

[2]University of New South Wales

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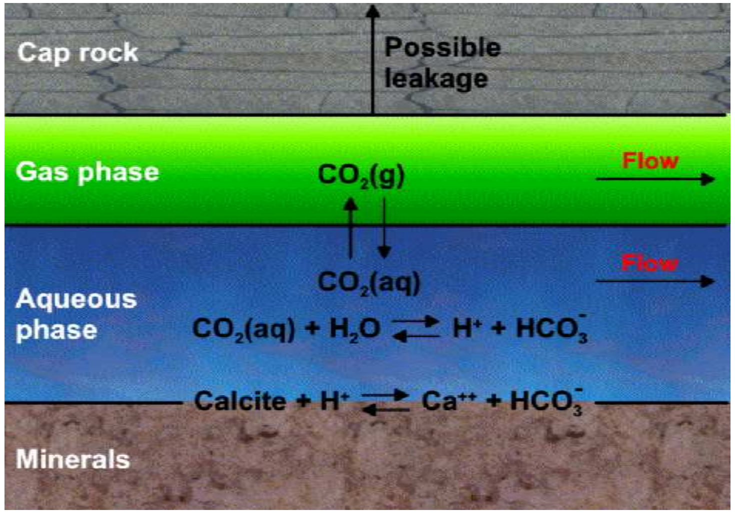
Outline

- Introduction
- Geomechanical properties of Rock
- Stress and strain
- Coupled simulation of fluid flow and geomechanics
- Case study

Introduction

Full-Physics Compositional Simulation

- Convective and dispersive flow
- Relative permeability hysteresis
- Gas solubility in aqueous phase
- Aqueous chemical equilibrium reactions
- Mineral dissolution and precipitation kinetics
- Vaporization of H_2O
- Predictions of brine density and viscosity
- Leakage through cap rock and thermal capability



CMG Training, 2008

Coupled Simulation of Fluid Flow and Geochemical Reaction

Material Balance Equation for CO_2

$$
\begin{aligned}
&\Delta T_g \, y_{CO_2,g} \, \Delta \Phi_g \; + \\
&\Delta T_w \left(y_{CO_2,aq} + y_{HCO_3^-,aq} + y_{CO_3^{--},aq} \right) \Delta \Phi_w \\
&+ r_x + q_{CO_2(g)} + q_{CO_2(aq)} + q_{HCO_3^-} + q_{CO_3^{--}} \\
&- \frac{V}{\Delta t} \Big[\left(N_{CO_2} + N_{HCO_3^-} + N_{CO_3^{--}} \right)^{n+1} - \\
&\qquad \left(N_{CO_2} + N_{HCO_3^-} + N_{CO_3^{--}} \right)^{n} \Big] = 0
\end{aligned}
$$

$$
N_{CO_2} = N_{CO_2(g)} + N_{CO_2(aq)}
$$

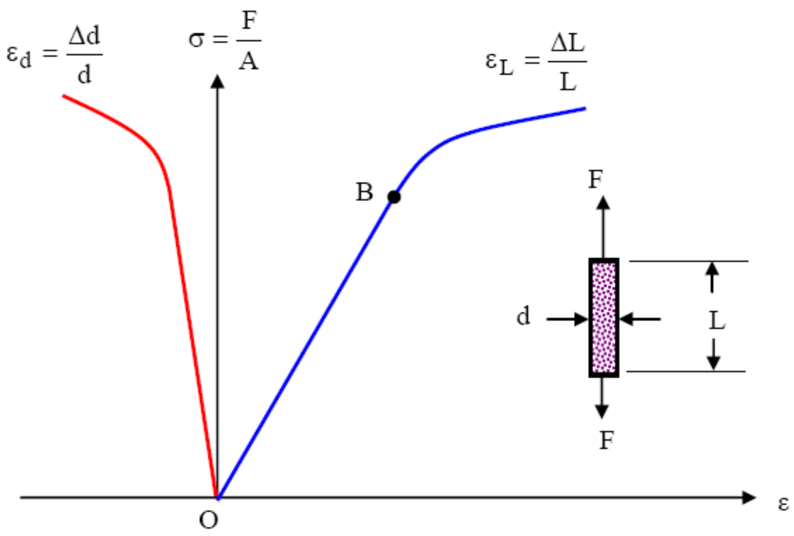
- Coupled simulation of fluid flow and geochemical reactions through the generation of compositional equation-of-state (EOS), which integrates the important geochemical simulations.

Introduction

- **Main impact from CO_2 injection:**
  - ✓Higher formation pressure due to CO2 injection
  - ✓CO_2 buoyancy force
- **Risk:**
  - ✓Destabilization of fault
  - ✓Leakage through cap rocks or wellbore
  - ✓Wellbore instability
- **Reservoir Characterization**
  - Orientation of minimum and maximum horizontal stress
  - Magnitude of minimum and maximum horizontal stress, pore pressure
  - Structural modelling:
  - Folding and unfolding, deformation, faulting, structural mapping

Geomechanical Property of Rock

- Tension and extension in a rod which is under axial tension and which is unrestricted laterally



Uniaxial Tension load

- **Young's modulus:**

$$E = \frac{\sigma}{\varepsilon_L}$$

- **Young's modulus:**
  - Ratio of lateral contraction to longitudinal extension

$$\nu = -\frac{\varepsilon_d}{\varepsilon_L}$$

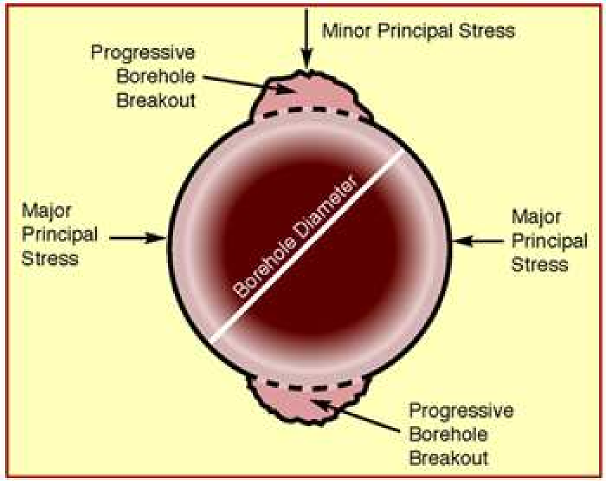
- **Bulk modulus:**

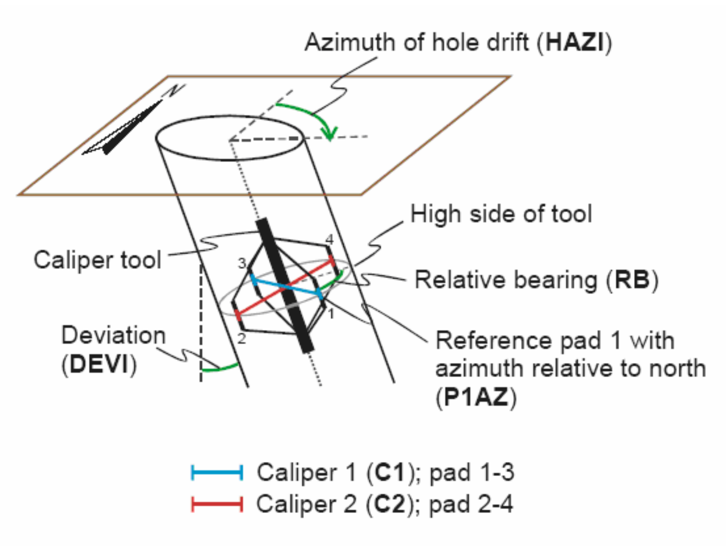
$$K = \frac{\text{hydrostatic pressure}}{\text{volumetric strain}}$$

$$K = \frac{E}{3(1-2\nu)}$$

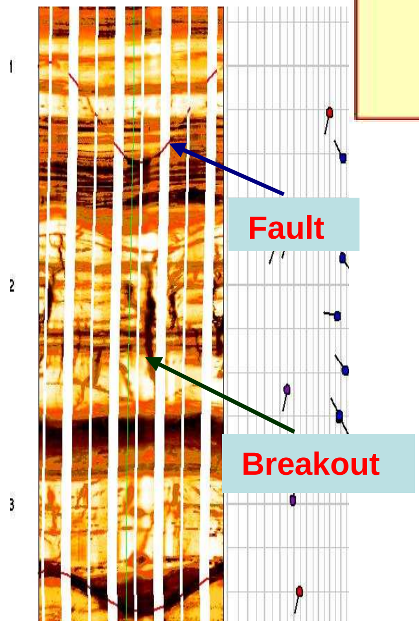
Orientation of Max and Min Horizontal Stresses

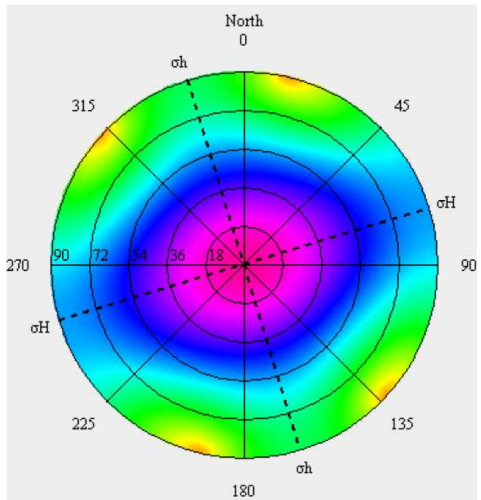
- ➢Borehole Breakouts
- ➢Drilling Induced Tensile Fractures
- ➢Earthquake Focal Mechanisms



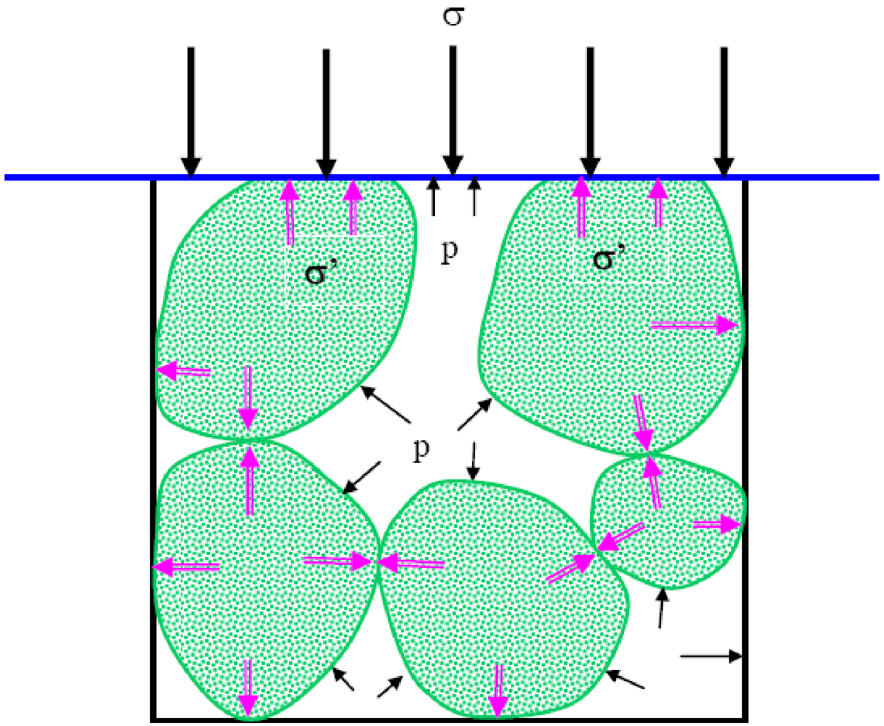


$$P1AZ = HAZI + a\tan\left(\frac{\tan(RB)}{\cos(DEV)}\right)$$





Pore Pressure, Effective stress and Total Stress



$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \alpha p \mathbf{I}$$ (In 3D)

$\boldsymbol{\sigma}$ – total stress

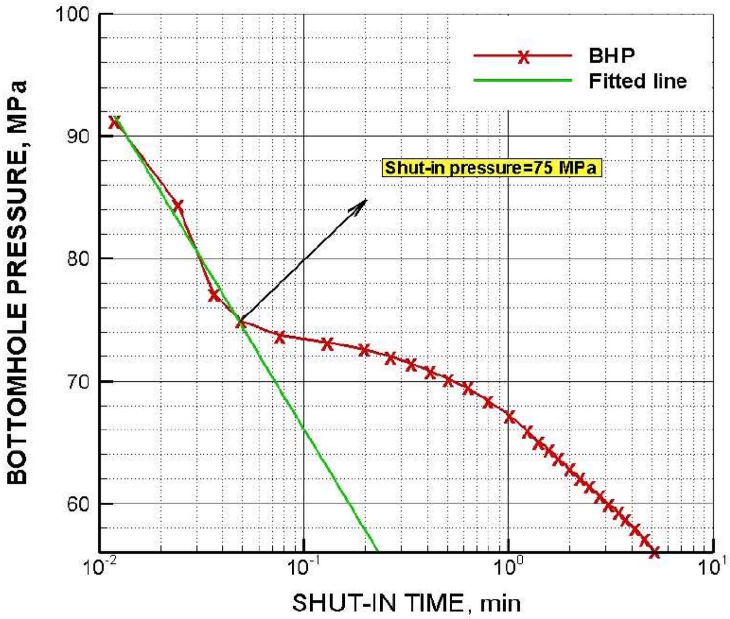
$\boldsymbol{\sigma}'$ – effective stress

p = pore pressure

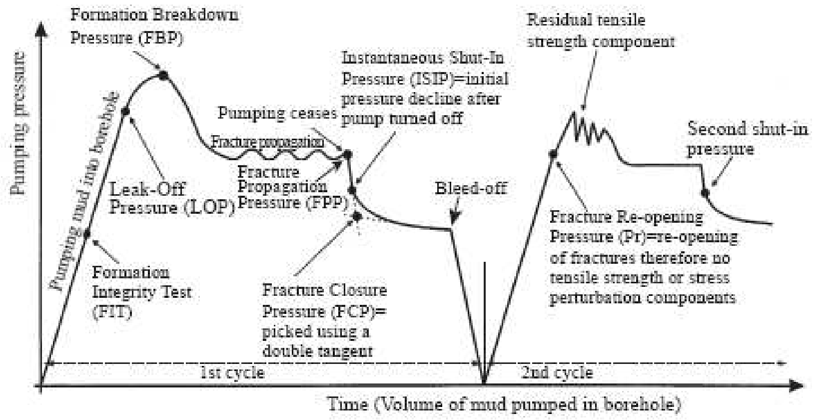
α - Biot's number

Minimum Horizontal Stress

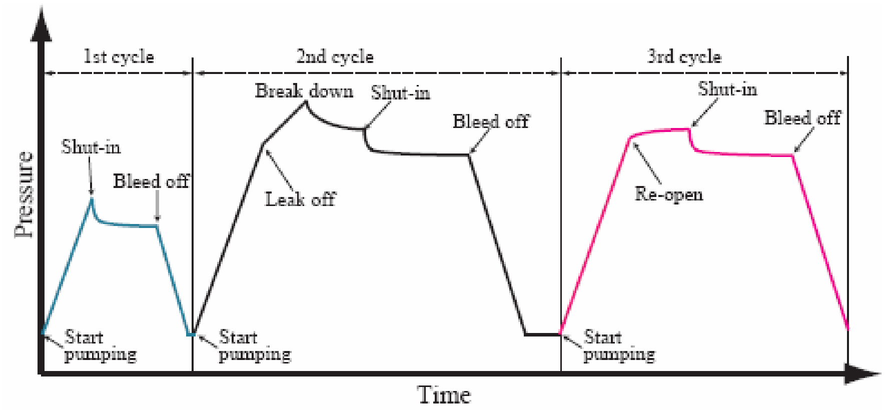
- ➢Post shut-in pressure analysis on mini-hydraulic fracturing data
- ➢Extended leak-off test (XLOT)



(Chen, 2009)



(White, et al., 2002)



(Weiren Lin, et al., 2008)

Vertical Stress

- Overburden stress or vertical stress, σ_v, at depth of D_s, with the average bulk density (*RHOB*, g/cc) and acceleration due to gravity, *g*:

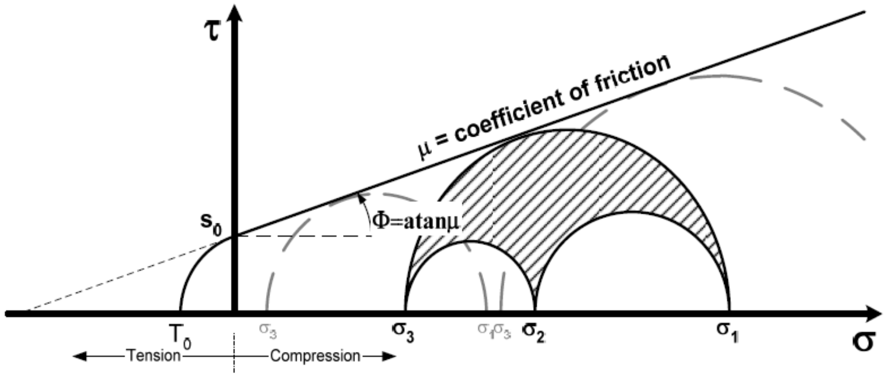
$$\sigma_v = \int_0^{D_s} RHOB \times g dD_s$$

- Trend line of RHOB:

$$RHOB = Ae^{B \cdot D_s}$$

A and *B* are the regression constants

Rock Frictional Strength



➢Rock principal stress vs internal friction:

$$\frac{\sigma_1 - P_p}{\sigma_3 - P_p} = f(\mu) = \left[\sqrt{1+\mu^2} + \mu\right]^2$$

σ_1 – maximum principal stress

σ_3 – minimum principal stress

P_p - pore pressure

μ – coefficient of friction

➢Normal Fault: $\sigma_v \geq \sigma_H \geq \sigma_h$

$$\frac{\sigma_1 - P_p}{\sigma_3 - P_p} = f(\mu) = \frac{\sigma_v - P_p}{\sigma_h - P_p}$$

$$\longrightarrow \quad \sigma_h = \frac{\sigma_v - P_p}{f(\mu)} + P_p$$

➢Strike-slip Fault: $\sigma_H \geq \sigma_v \geq \sigma_h$

$$\frac{\sigma_1 - P_p}{\sigma_3 - P_p} = f(\mu) = \frac{\sigma_H - P_p}{\sigma_h - P_p}$$

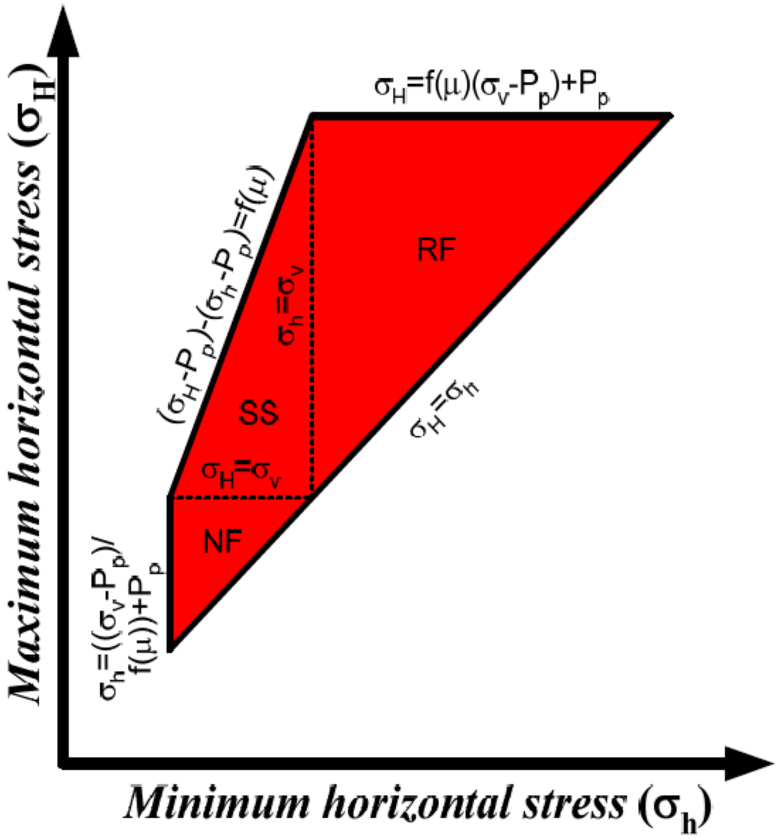
➢Reverse Fault: $\sigma_H \geq \sigma_h \geq \sigma_v$

$$\frac{\sigma_1 - P_p}{\sigma_3 - P_p} = f(\mu) = \frac{\sigma_H - P_p}{\sigma_v - P_p}$$

or

$$\sigma_H = (\sigma_v - P_p) \cdot f(\mu) + P_p$$

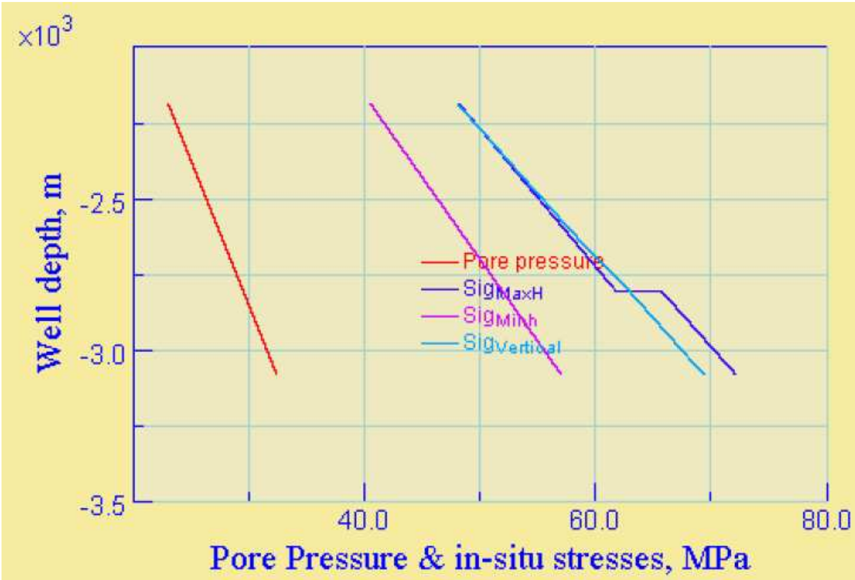
Internal Friction for Three Different Faulting Regimes



- **Normal Fault:** $\sigma_v \geq \sigma_H \geq \sigma_h$
- **Strike-slip Fault:** $\sigma_H \geq \sigma_v \geq \sigma_h$
- **Reverse Fault:** $\sigma_H \geq \sigma_h \geq \sigma_v$

- To further constrain the horizontal stress:
  - Wellbore breakout angle (FMI, BHTV, etc.)
  - Rock compressive strength

Example of Pore Pressure and Stress



Well depth (m):	3083.509
Inlination (deg):	0.0
Azimuth (deg):	0.0
Max horizontal stress (MPa):	72.154
Min horizontal stress (MPa):	57.045
Vertical stress (MPa):	69.439
Azimuth of Min-h stress (deg):	343.2
Pore pressure (MPa):	32.439
Poisson's ratio:	0.316
Friction angle (deg):	46.787
Biot coefficient:	1.0
Cohesive strength (MPa):	5.378
Tensile strength (MPa):	2.263

(Chen, 2009)

Compressive and Shear Wave Slowness

(Well Log Data)

❖**Shear modulus:**

$$G = 13474.45 \times \frac{RHOB}{DTS^2}$$

❖**Bulk modulus:**

$$Kbulk = 13474.45 \times RHOB \times \left(\frac{1}{DT^2}\right) - \frac{4G}{3}$$

❖**Poisson's Ratio:**

$$\nu = \frac{3 \times Kbulk - 2G}{6 \times Kbulk + 2G}$$

❖**Young's modulus:**

$$E = \frac{9 \times G \times Kbulk}{3 \times Kbulk + G}$$

❖**Bulk compressibility:**

$$C_b = 1000 \times RHOB \times \left(\frac{1}{DT^2} - \frac{4}{3 \times DTS^2}\right)$$

❖**Internal frictional angle:**

$$\phi = \phi_{shale} \cdot VSH + \phi_{sandsone} \cdot (1 - VSH)$$

❖**Unconfined compressive strength:**

$$UCS = 1.35 \cdot \left(\frac{304.8}{DT}\right)^{2.75} \cdot VSH + 1200 \cdot e^{-0.0313 \cdot DT} \cdot (1 - VSH)$$

❖**Cohesive strength:**

$$S_0 = \frac{UCS}{2} \cdot \frac{1 - \sin\phi}{\cos\phi}$$

❖**Tensile strength:**

$$T_0 = \frac{UCS}{12}$$

Al-Qahtani et al, 2001

❖**Static geomechanical property:**

❖**Linear regressioned from dynamic property**

RHOB = bulk density log (g/cc)

DTS - Shear wave slowness (μμs/ft

DT - compressional wave slowness (μμs/ft

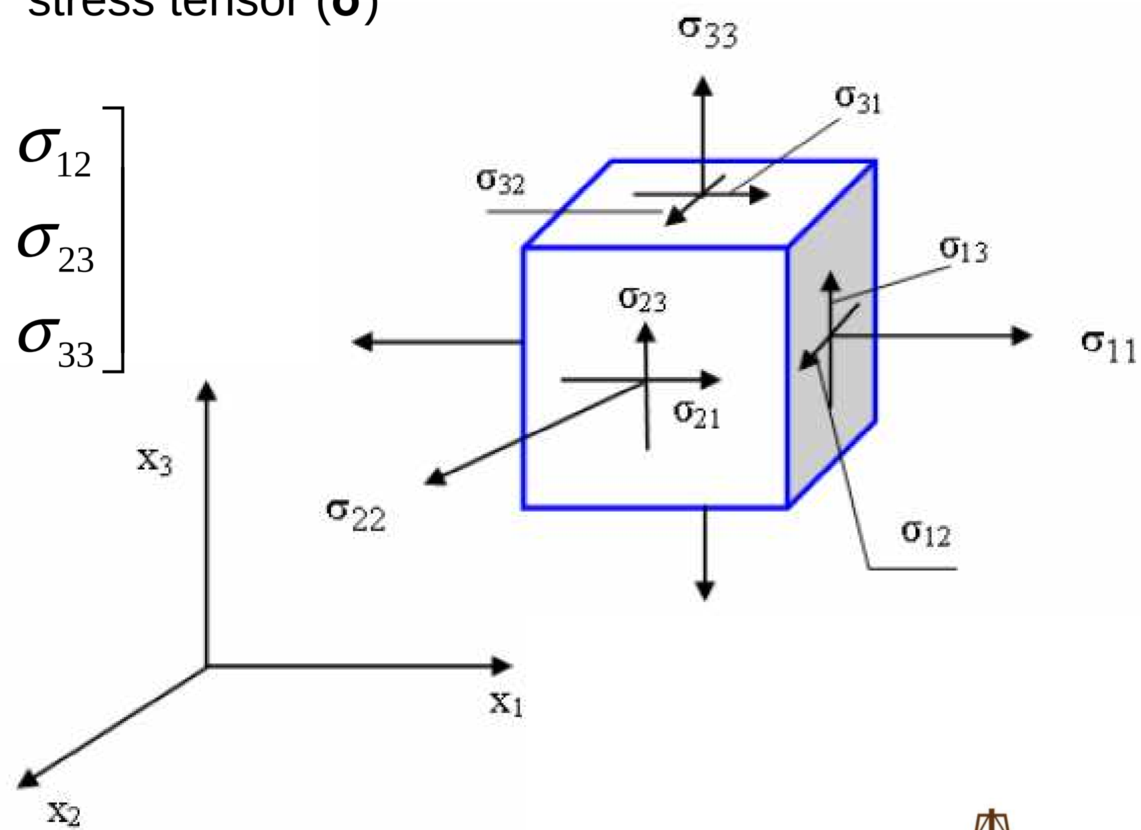
VSH - volume fraction of shale

Stress Tensor

- Traction Force per Unit (**T**)=

  Unit normal vector (**n**) × stress tensor (**σ**)

$$\boldsymbol{\sigma} = \sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$



Mean & Principal Effective Stress

- Mean effective stress:

$$\sigma_m = \frac{1}{3}\boldsymbol{\sigma}'_{ii} = \frac{1}{3}\left(\sigma'_{11} + \sigma'_{22} + \sigma'_{33}\right)$$

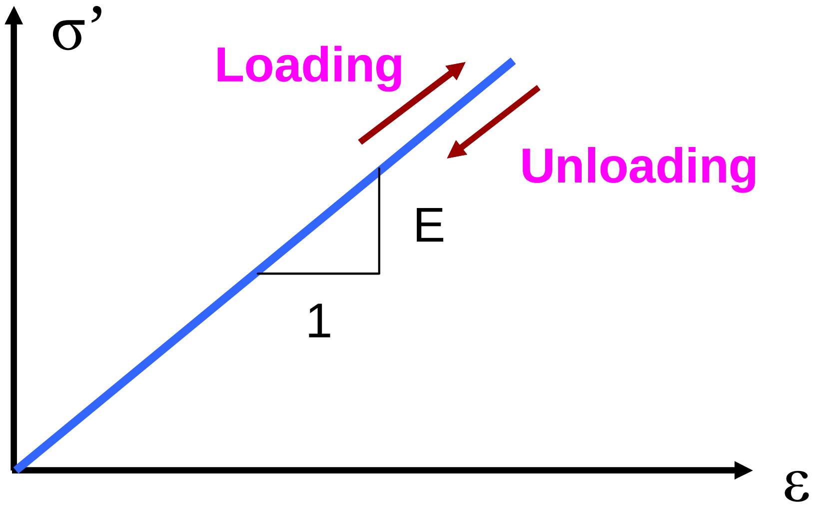
- Principal effective stress:

$$\sigma'_{ij} = \begin{bmatrix} \sigma'_1 & 0 & 0 \\ 0 & \sigma'_2 & 0 \\ 0 & 0 & \sigma'_3 \end{bmatrix}$$

$$\text{Assume that}: \sigma'_1 > \sigma'_2 > \sigma'_3$$

Constitutive Laws

➢Linear elasticity: Loading and unloading have the same stress path

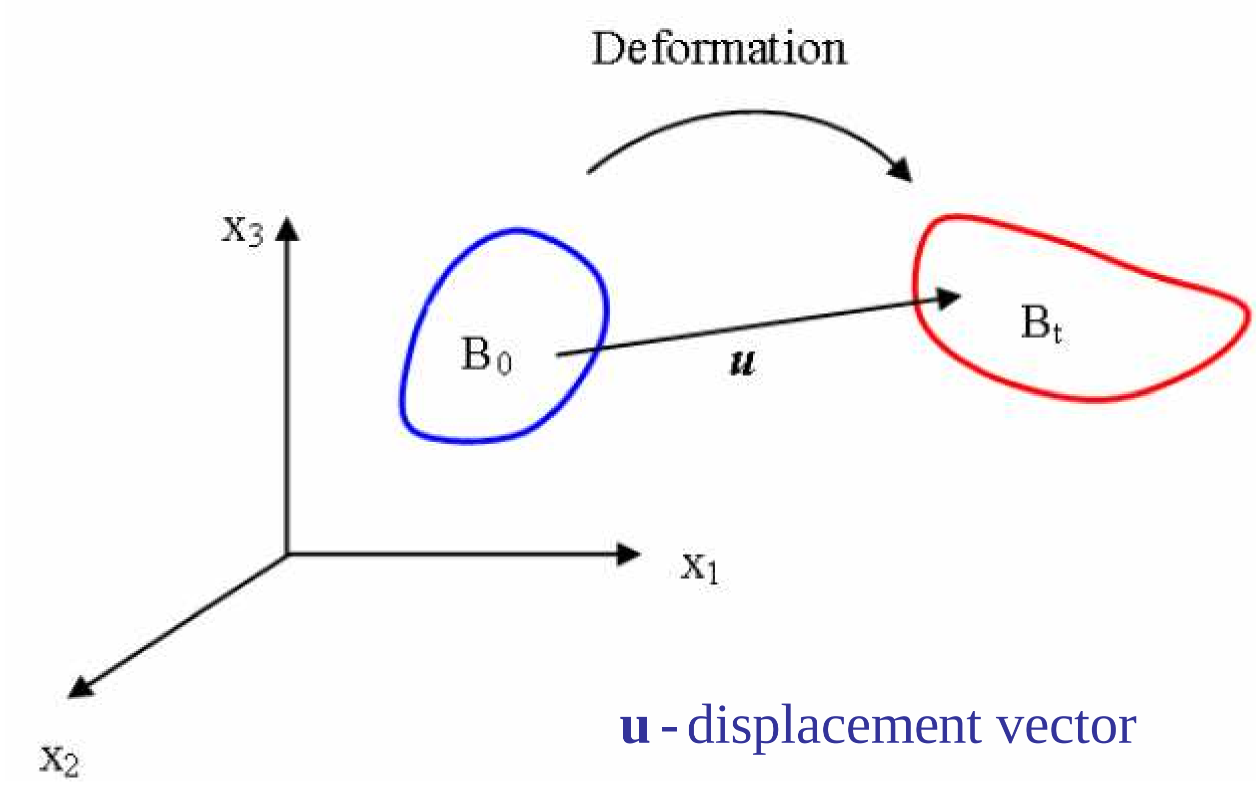


Linear Elastic Model

σ'	:	Effective stress
ε	:	Strain
E	:	Young's modulus

Displacement & Deformation

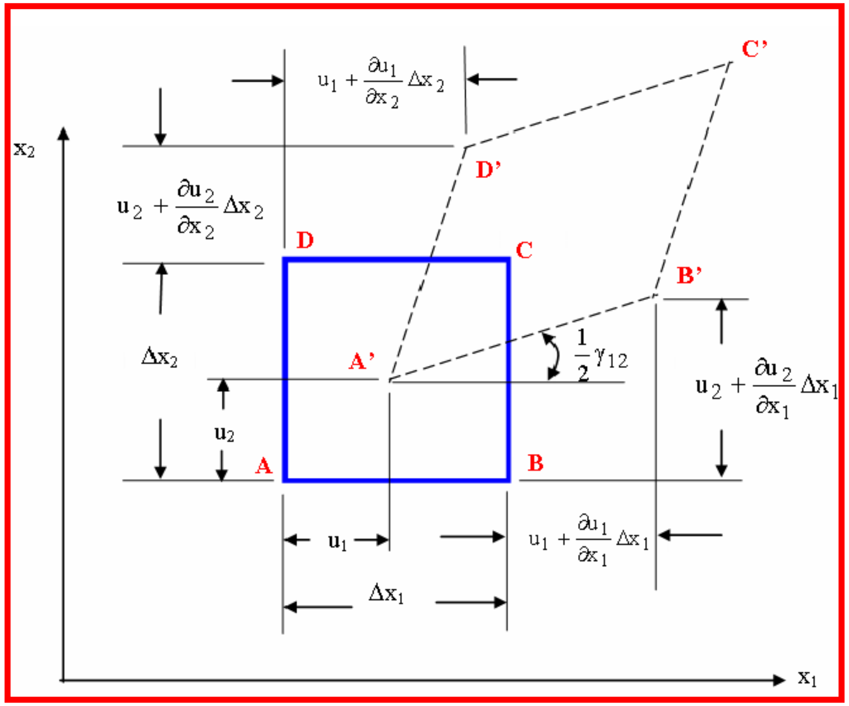
- Changing both the shape and the location:



u - displacement vector

Bt - deformed configuration

Strain



$$\boldsymbol{\varepsilon} = \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

- Normal Strain:

$$\varepsilon_{11} = \lim_{\Delta x_1 \to 0} \frac{A'B' - AB}{AB} = \lim_{\Delta x_1 \to 0} \frac{\left[\Delta x_1 + \left(\partial u_1 / \partial x_1\right)\Delta x_1\right] - \Delta x_1}{\Delta x_1}$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

- Shear Strain:

$$\gamma_{12} = \lim_{\substack{\Delta x_1 \to 0 \\ \Delta x_2 \to 0}} \left\{ \frac{\pi}{2} - \angle D'A'B' \right\}$$

$$= \lim_{\substack{\Delta x_1 \to 0 \\ \Delta x_2 \to 0}} \left\{ \frac{\pi}{2} - \left\{ \frac{\pi}{2} - \frac{\left(\partial u_2 / \partial x_1\right)\Delta x_1}{\Delta x_1} - \frac{\left(\partial u_1 / \partial x_2\right)\Delta x_2}{\Delta x_2} \right\} \right\}$$

$$\gamma_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}$$

$$\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\gamma_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

Volumetric Strain

$$\text{Volumetric Strain} = \frac{\text{change in volume}}{\text{initial volume}}$$

$$\varepsilon_v = \varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

Absolute Permeability

- Matrix Permeability

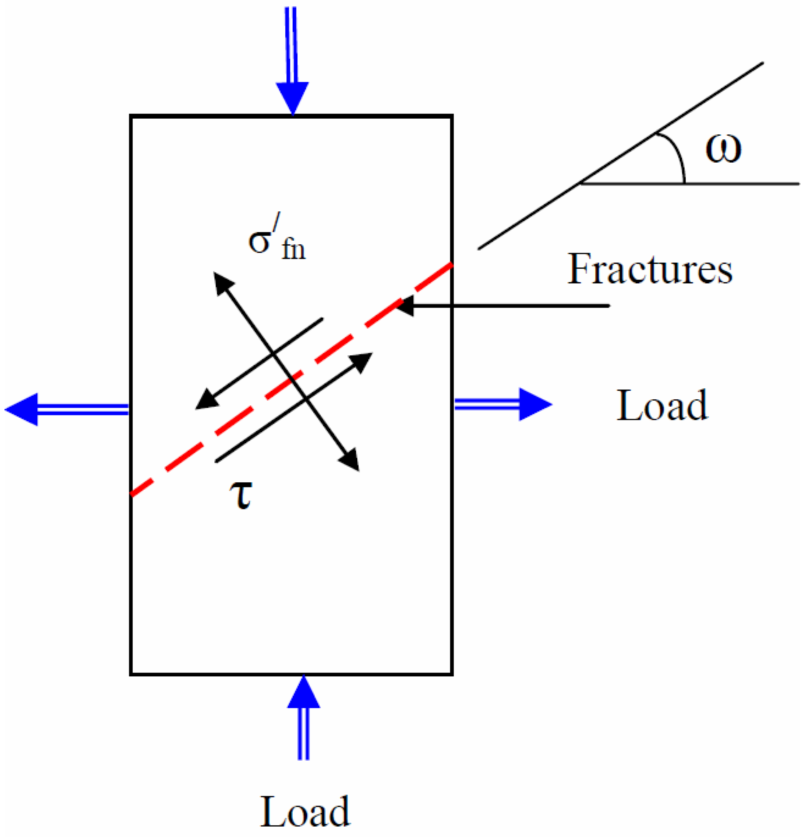
    - Empirical formula (Li and Chalaturnyk)
    - Look-up Table

- Fracture Permeability

    - Barton-Bandis Model (BB Model)

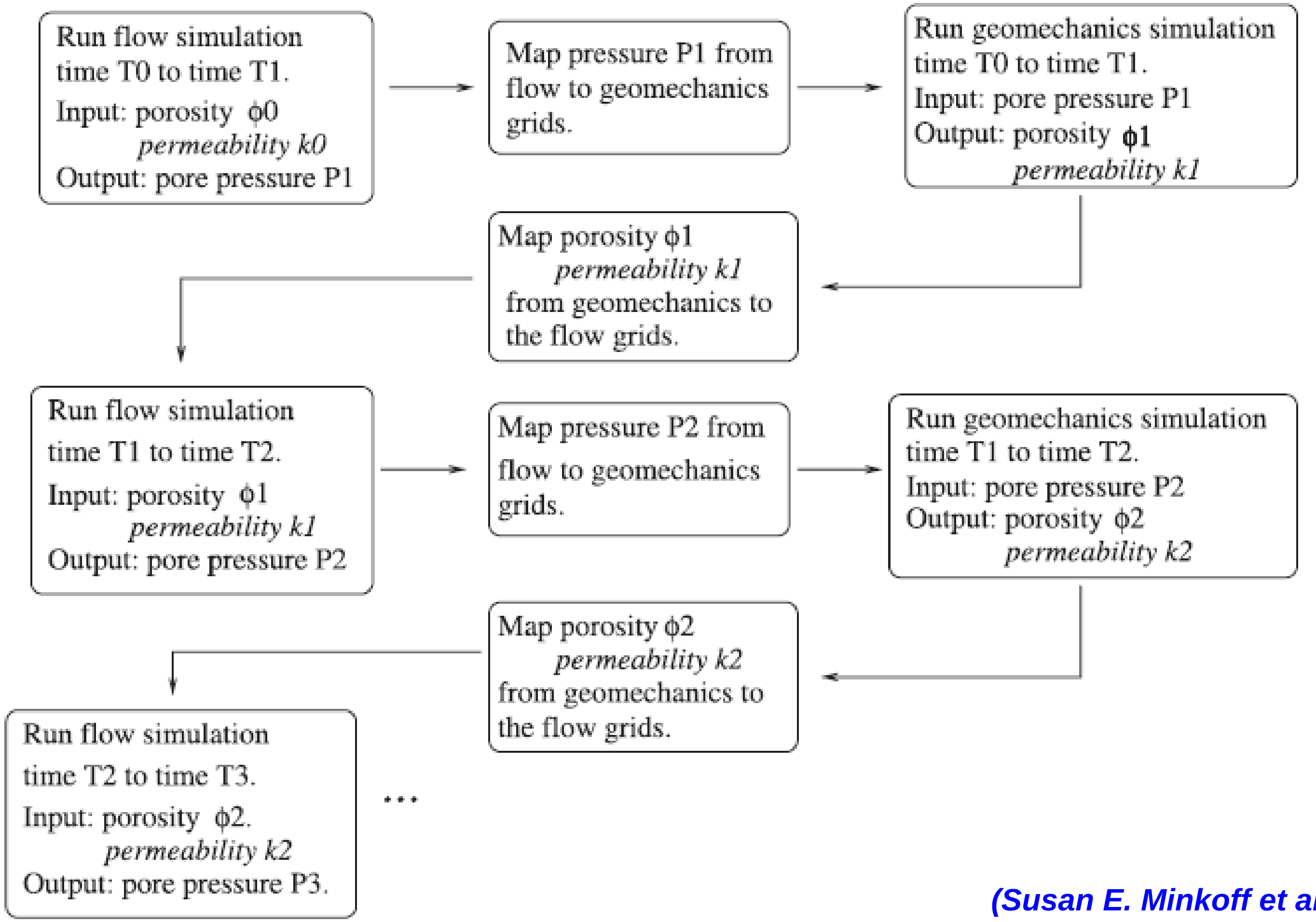
Barton-Bandis Model

- A secondary fracture system is defined in the grid via dual-permeability
- As pressure increase in the regular grid the stresses are altered, causing the normal stresses on the fractures to increase.
- Eventually the Stress breaks past the Failure Envelope of the rock, causing a fracture to appear (open) and allow fluids to pass through.



(CMG, 2009)

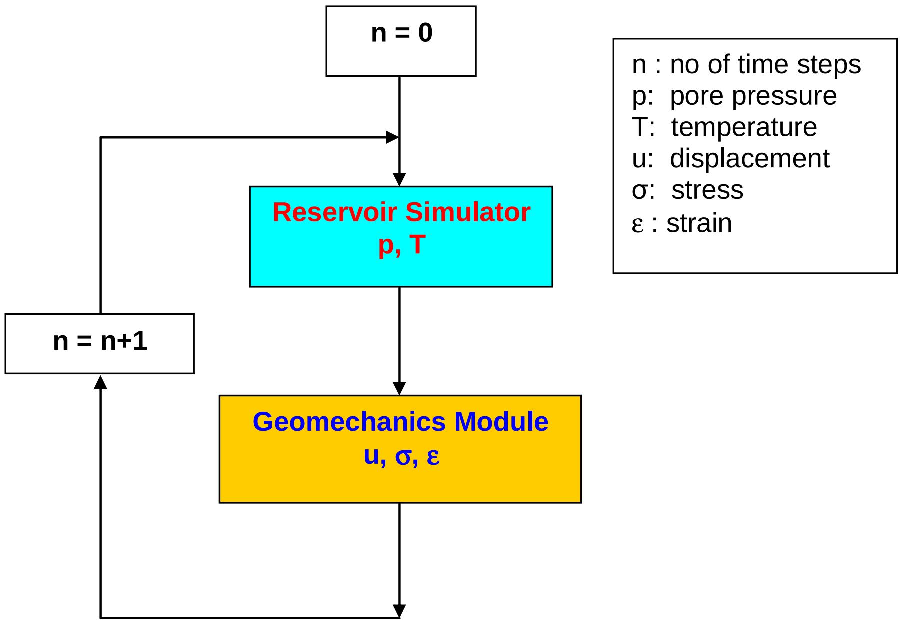
Loose Coupling Algorithm



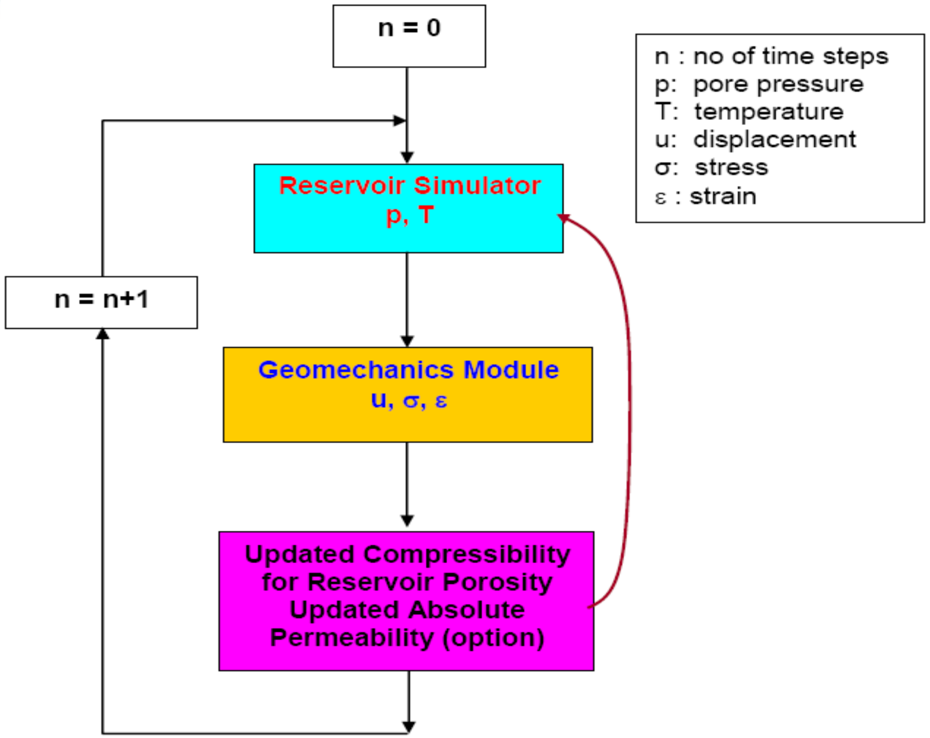
(Susan E. Minkoff et al., 2003)

Geomechanical Simulation Coupled with Compositional Simulator

- **Finite element approach:**



One Way Coupling Simulation

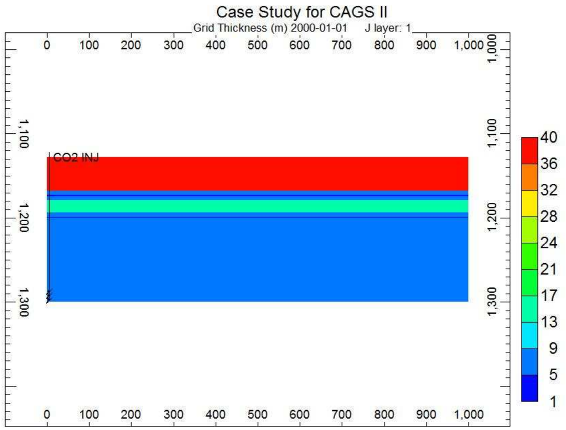


Two Way Coupling Simulation

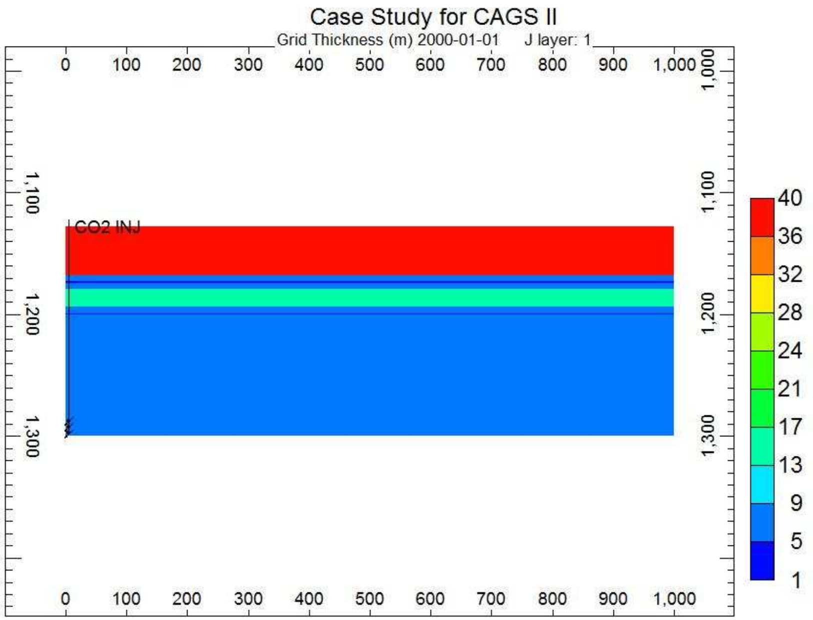
(CMG, 2009)

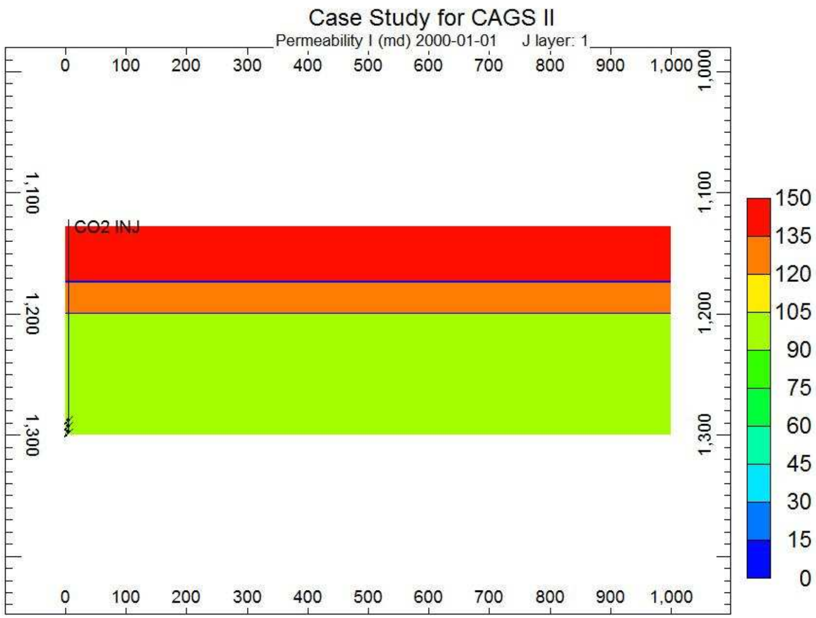
Case Study
Leakage Risk of Caprock

- **Two-way coupled simulation:**
  - Grid Dimension: 2m×10m (horizontal)
  - Grid Number: 500×1×27
  - Porosity: 0.18
  - Kv/Kh=1
  - Sgrm = 0.3
  - Injection Well: (3, 1, 1)
  - Perforation Interval: (3, 1, 25) to (3,1, 27)
  - Injection Rate: 1×10^4 m^3/day (STG surface gas rate)
  - Injection Period: 2000-1-1 to 2003-1-1
  - Simulation Period: 2000-1-1 to 2200-1-1

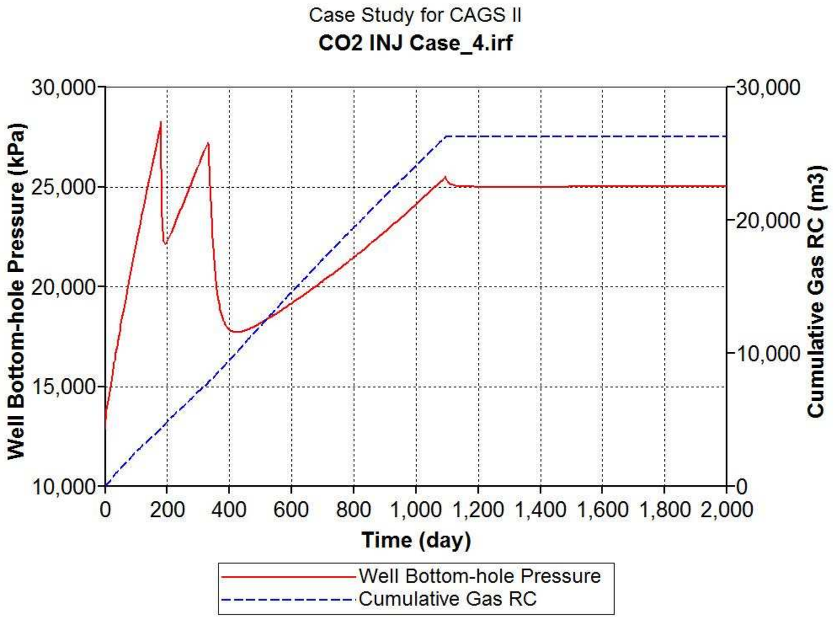


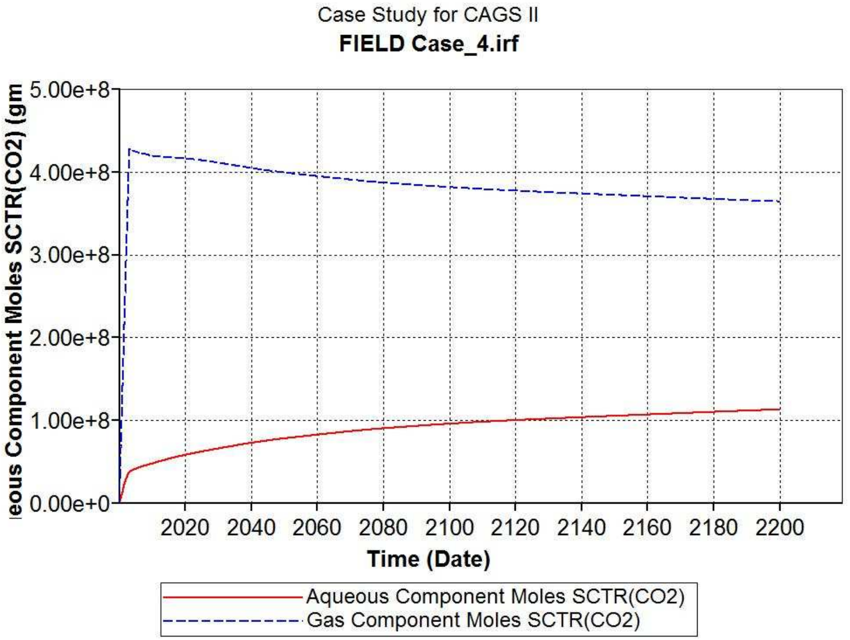
Permeability Model





Results-200yrs Later





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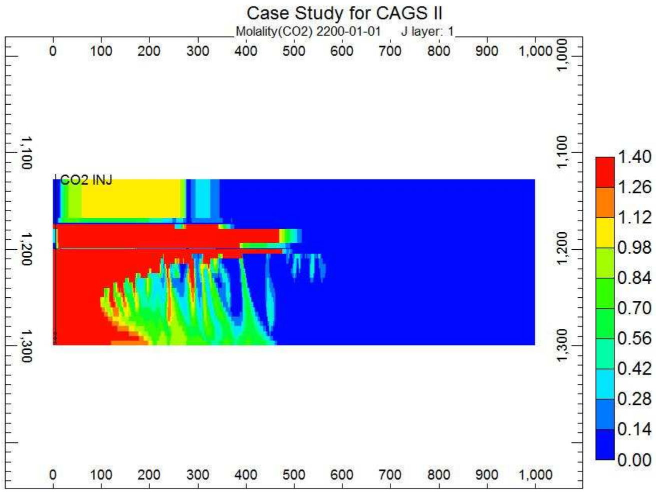

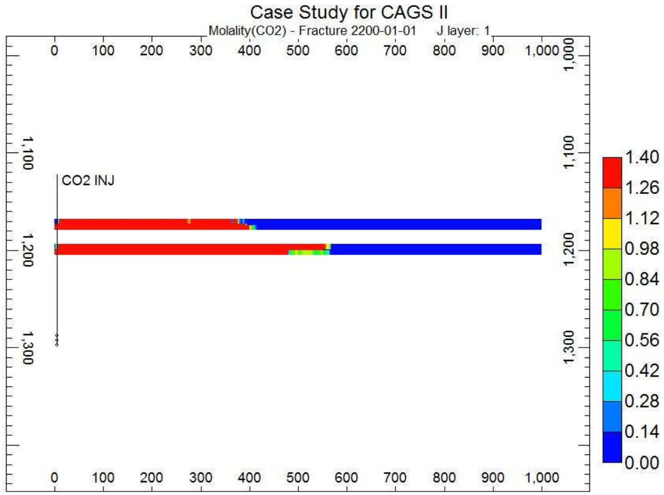

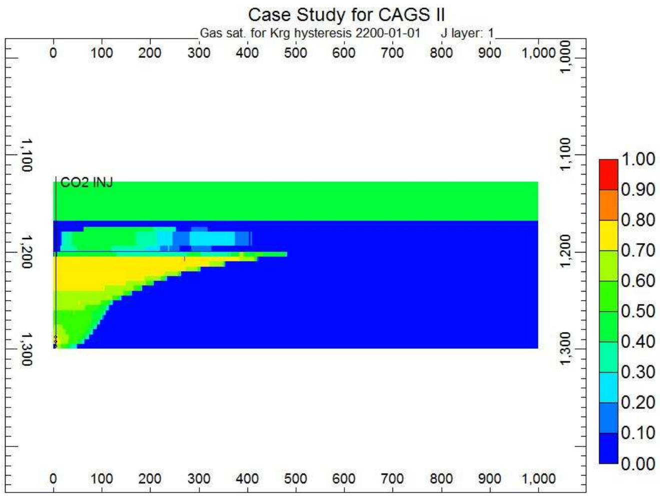

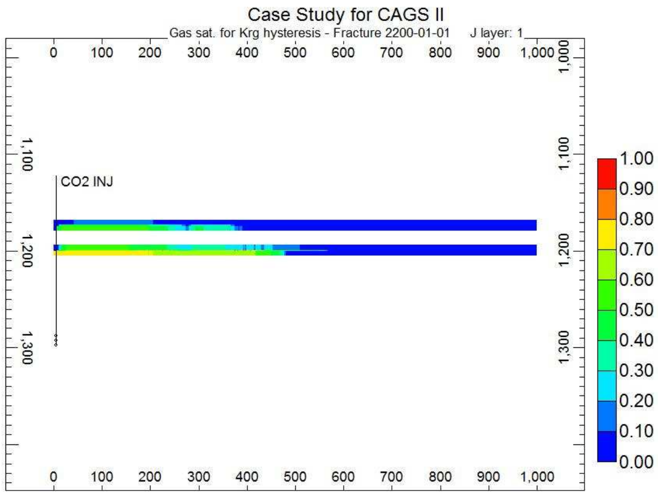

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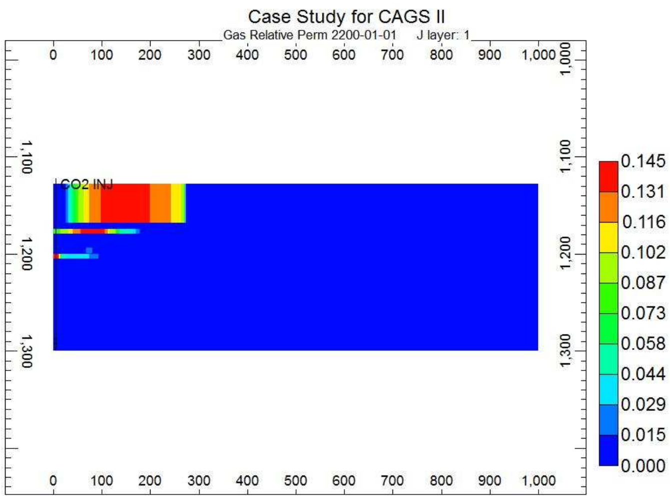

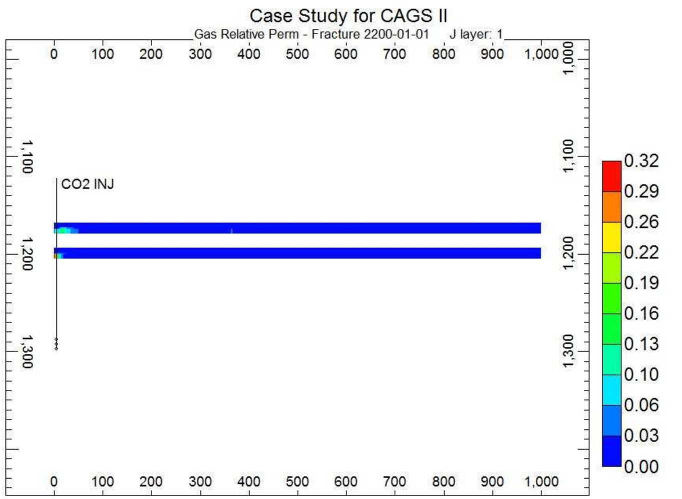

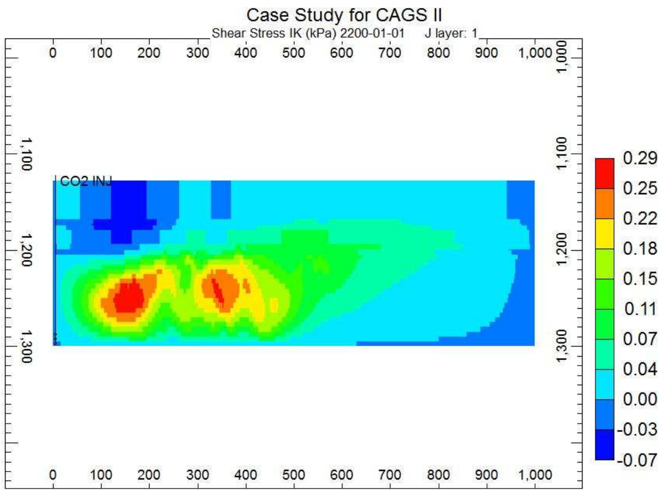

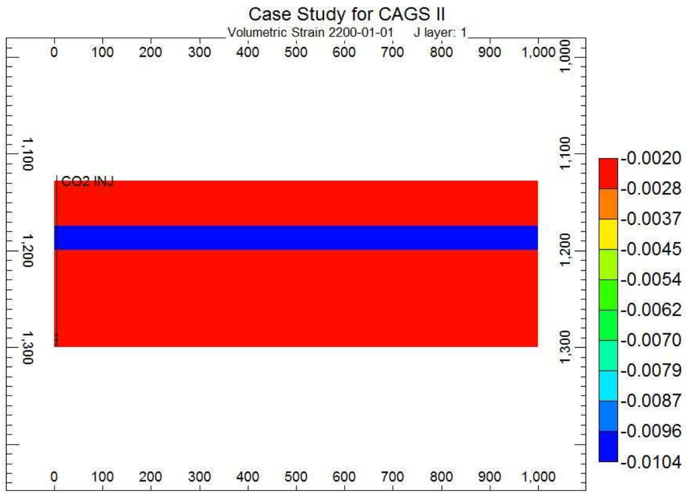

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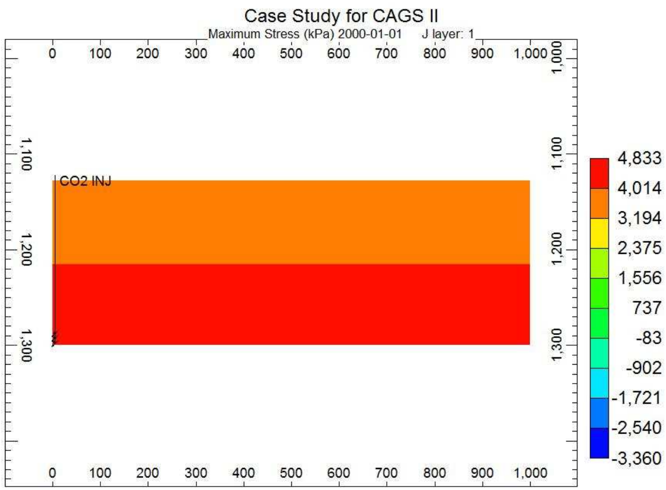
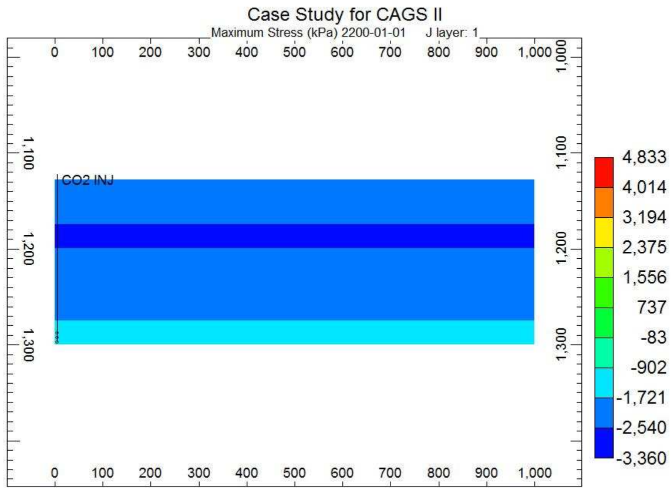
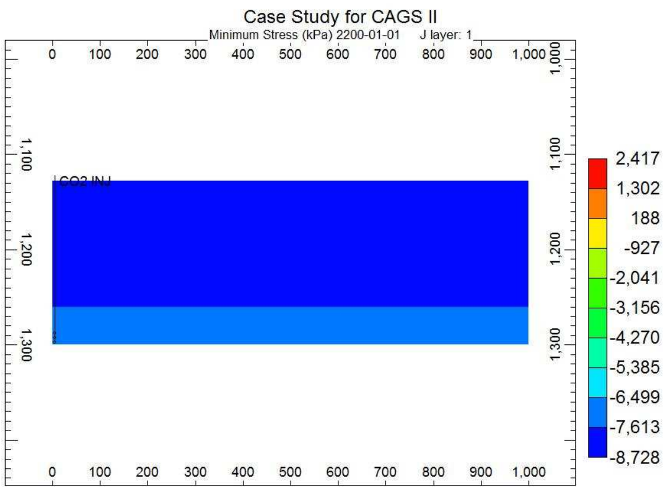
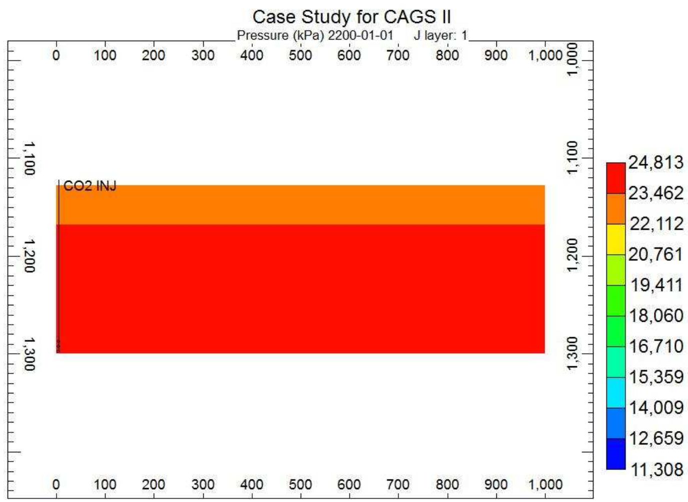




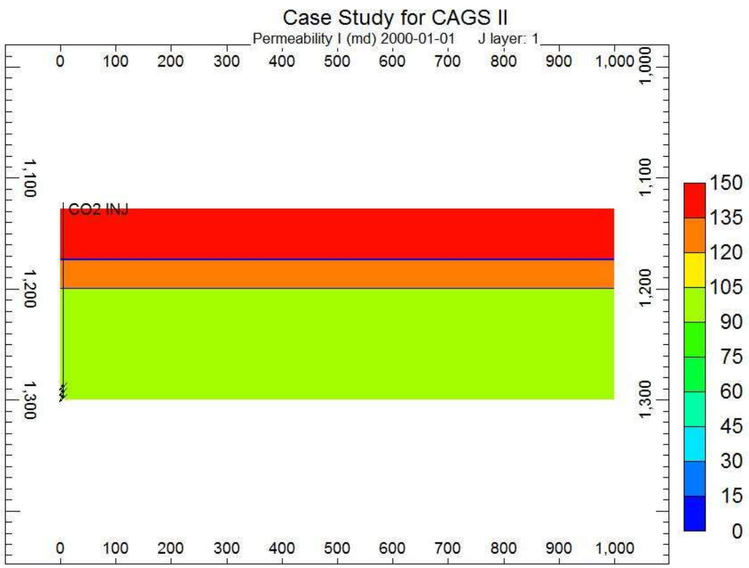


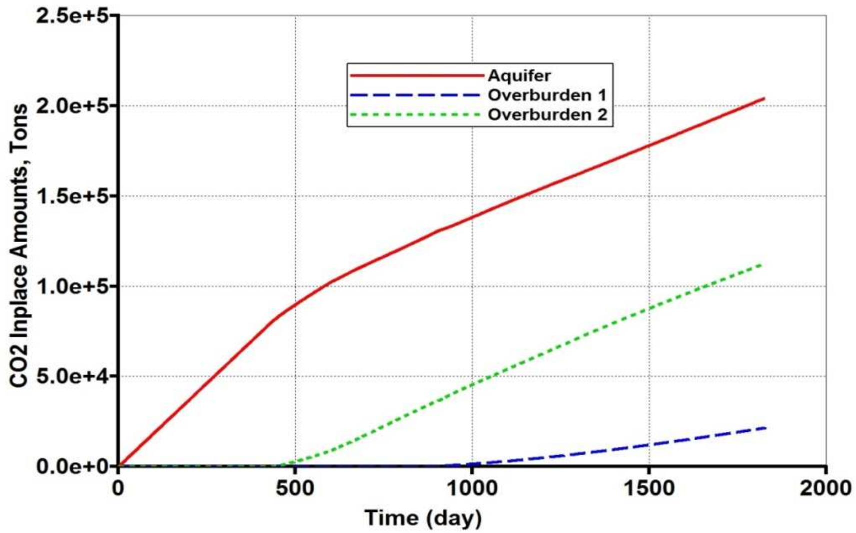


Results-200yrs Later

Total Cum Inj, mol = 4.65464E+08

CO2 Storage Amounts in Reservoir	Moles	kg
Gaseous Phase	= 0.00000E+00	0.00000E+00
Supercritical Phase	= 4.09517E+08	1.80228E+07
Trapped due to Hysteresis	= 1.54067E+08	6.78048E+06
Dissolved in Water	= 6.79868E+07	2.99210E+06





(CMG, 2009)

Summary

- ➢Coupled numerical simulation of fluid flow and geomechanics is based on the detailed reservoir characterisation of structure, petrophysical property and geomechanical property, ect.

- ➢Coupled simulation can improve our understandings of both movement of CO2 plume and change of geomechanical pattern.

- ➢Besides the effective storage capacity assessment, the coupled simulation can provide the risk information of leakage.